(D1) **Dust and Young Stars in Galaxies** **[75 marks]**

As a by-product of the star-forming process in a galaxy, interstellar dust can significantly absorb stellar light in ultraviolet (UV) and optical bands, and then re-emit in far-infrared (FIR), which corresponds to a wavelength range of 10-300 μm.

1.1. In the UV spectrum of a galaxy, the light is majorly contributed by the young stellar population generated in the recent star-formation process, thus the UV luminosity could act as a reliable tracer of star-formation rate (SFR) of galaxy. Since the observed UV luminosity is strongly affected by dust attenuation, extragalactic astronomers define an index called *UV continuum slope* (β) to quantify the shape of the UV continuum:

$$f_\lambda = B \cdot \lambda^\beta$$

where f_λ is the monochromatic flux of the galaxy at given wavelength λ (in the unit of $\mathrm{J s^{-1} m^{-3}}$) and B is a scaling constant.

(D1.1.1) (6 points) The AB magnitude is defined as:

$$\mathrm{m_{AB}} = -2.5 \log \frac{f_\nu}{3631\ \mathrm{Jy}}, 1\ \mathrm{Jy} = 10^{-26}\ \mathrm{J\ s^{-1}\ Hz^{-1}\ m^{-2}}$$

The AB magnitude of a typical galaxy is roughly constant in the UV band. What is the **UV slope** of such kind of galaxy? (Hint: $f_\nu \Delta\nu = f_\lambda \Delta\lambda$)

(D1.1.2) (12 points) Table 1 presents the UV photometry results for a $z = 6.60$ galaxy called *CR7*. **Plot** the AB magnitude of *CR7* versus rest-frame wavelength (in logarithm scale) on the graph paper as **Figure 1**.

(D1.1.3) (5 points) Calculate *CR7*'s UV slope using the method of least squares, **plot** the best-fit UV continuum on *Figure 1* and make a comparison with the results you obtained in (D1.1.1). Is it dustier than the typical galaxy in (D1.1.1)? **Please answer with [YES] or [NO]**. (Hint: Express $\mathrm{m_{AB}}$ as a function of λ and m_{1600} ,where m_{1600} is the AB magnitude at $\lambda_0 = 160\mathrm{nm}(1600\ \text{Å})$)

Table 1. UV Photometry of CR7 at z = 6.60

Band	*Y*	*J*	*H*	*K*
Central Wavelength (μm)	1.05	1.25	1.65	2.15
AB Magnitude	24.71 ± 0.11	24.63 ± 0.13	25.08 ± 0.14	25.15 ±0.15

1.2. Under the assumption that dust grains in galaxy absorb the energy of UV photon and re-emit it by blackbody radiation, the relation between UV continuum slope, UV brightness and FIR brightness could be established:

$$\mathrm{IRX} \equiv \log\left(\frac{F_{FIR}}{F_{1600}}\right) = f(\beta)$$

where F_{FIR} is the observed far-infrared flux and F_{1600} is the observed flux at 160nm(1600 Å) (defined as $F_\lambda = \lambda \cdot f_\lambda$). Table 2 presents 20 measurements of β, F_{FIR} and F_{1600} in nearby galaxies (Meurer et al. 1999).

Table 2. UV slope, flux and FIR flux of 20 nearby galaxies

Galaxy Name	UV Slope β	$\log F_{1600}$ (10^{-3}Js^{-1}m^{-2})	$\log F_{FIR}$ (10^{-3}Js^{-1}m^{-2})
NGC4861	-2.46	-9.89	-9.97
Mrk 153	-2.41	-10.37	-10.92
Tol 1924-416	-2.12	-10.05	-10.17
UGC 9560	-2.02	-10.38	-10.41
NGC 3991	-1.91	-10.14	-9.8
Mrk 357	-1.8	-10.58	-10.37
Mrk 36	-1.72	-10.68	-10.94
NGC 4670	-1.65	-10.02	-9.85
NGC 3125	-1.49	-10.19	-9.64
UGC 3838	-1.41	-10.81	-10.55
NGC 7250	-1.33	-10.23	-9.77
NGC 7714	-1.23	-10.16	-9.32
NGC 3049	-1.14	-10.69	-9.84
NGC 3310	-1.05	-9.84	-8.83
NGC 2782	-0.9	-10.5	-9.33
NGC 1614	-0.76	-10.91	-8.84
NGC 6052	-0.72	-10.62	-9.48
NGC 3504	-0.56	-10.41	-8.96
NGC 4194	-0.26	-10.62	-8.99
NGC 3256	0.16	-10.32	-8.44

(D1.2.1) (14 points) Based on the data given in Table 2, **plot** the IRX $-\ \beta$ diagram on the graph paper as **Figure 2** and find a linear fit to the data. **Write down** your best-fit equation (i.e. IRX $= a \cdot \beta + b$) by the side of your diagram.

(D1.2.2) (6 points) Quantify the **dispersion** (in unit of dex, $\mathrm{dex}(x) = 10^x$) between the observed and expected IRX($\widehat{\mathrm{IRX}}$) using the following equation:

$$\sigma = \sqrt{\frac{\sum(\Delta \mathrm{IRX}_i)^2}{N-1}}\ (\text{unit: dex})\ where\ \Delta \mathrm{IRX}_i = \mathrm{IRX}_i - \widehat{\mathrm{IRX}}_i$$

1.3. Under previous assumption of energy transfer process, the ratio of F_{FIR} to F_{1600} could be expressed as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1$$

Where F_{1600} is unattenuated flux, A_λ is the dust absorption in magnitude as a function of wavelength λ.

(D1.3.1) (6 points) Express A_{1600} as a function of IRX.

(D1.3.2) (12 points) Based on Table 2 data and the function of $A_{1600}(IRX)$ you derived above, **plot** the $A_{1600} - \beta$ diagram on the graph paper as **Figure 3** and find a linear fit to the data. **Write down** your best-fit equation (i.e.$A_{1600} = a' \cdot \beta + b'$) by the side of your diagram.

(D1.3.3) (2 points) If your linear model in (D1.3.2) is correct, what is the expected **UV continuum slope** β_0 of a dust-free galaxy?

1.4. After establishing the local relation between UV continuum slope and IRX, we could probably test this empirical law in the high-redshift universe. In 2016, researchers obtained an Atacama Large Millimeter/submillimeter Array (ALMA) observation of CR7, and the FIR continuum was undetected with a 3σ upper limit of 21 μJy at rest-frame wavelength of 160 μm (Matthee et al. 2017), which corresponds to an FIR flux of 1.5×10^{-19} J/s/m^2.

(D1.4.1) (6 points) Calculate the **IRX of *CR7***. Is it an upper limit or lower limit?

Hint: here F_{1600} should be written in the form of:

$$F_{1600} = (1+z) \cdot \lambda_0 \cdot f_{\lambda_0 \cdot (1+z)}$$

where $\lambda_0 = 160nm(1600\ \text{Å})$ *and* $f_{\lambda_0 \cdot (1+z)}$ *is observed flux at rest-frame*

(D1.4.2) (6 points) Is the current observation deep enough to show any deviation of *CR7* from IRX$-\beta$ relation you just derive in the local universe? **Please answer with [YES] or [NO]**.

Solution:

1.1. UV continuum slope (β)[23 marks]

(D1.1.1) [6 marks] The AB magnitude of a typical galaxy is roughly constant, therefore:

$$f_\nu = const. \dots\dots (1)$$

Conversion between f_ν and f_λ could be expressed as:

$$f_\nu \Delta\nu = f_\nu \left(\frac{c}{\lambda - \Delta\lambda} - \frac{c}{\lambda}\right) = \frac{f_\nu c}{\lambda^2}\Delta\lambda = f_\lambda \Delta\lambda \Rightarrow f_\lambda \propto \lambda^{-2} \dots\dots (2)$$

Therefore, the UV continuum slope should be $\beta = -2$.

[Equation (2): 3 marks; final result: 3 marks]

(D1.1.2) [12 marks]First of all, we need to convert all the wavelength into rest-frame value, using the definition of redshift:

$$z = \frac{\lambda_{obs}}{\lambda_{rest}} - 1 \Rightarrow \lambda_0 = \frac{\lambda_{obs}}{1+z} \dots\dots (3)$$

Thus, the rest-frame wavelength of four bands are listed in the following table:

Band	*Y*	*J*	*H*	*K*
Rest-frame Wavelength (Å)	1382	1645	2171	2829

The plot is shown as *Figure 1*. Detailed standards of grading the plot are in the solution of (D1.1.3).

[Equation (3): 2 marks; Correct wavelengths: 2 marks; Figure: 8 marks]

(D1.1.3)[5marks]Here we can express the AB magnitude of CR7 (UV continuum only) using β and λ_{rest}:

$$m_{AB} = -2.5\log f_\nu + C = -2.5\log(\lambda^2 f_\lambda) + C = -2.5(\beta+2)\log\left(\frac{\lambda}{1600\ \text{Å}}\right) + m_{1600} \ldots\ldots (4)$$

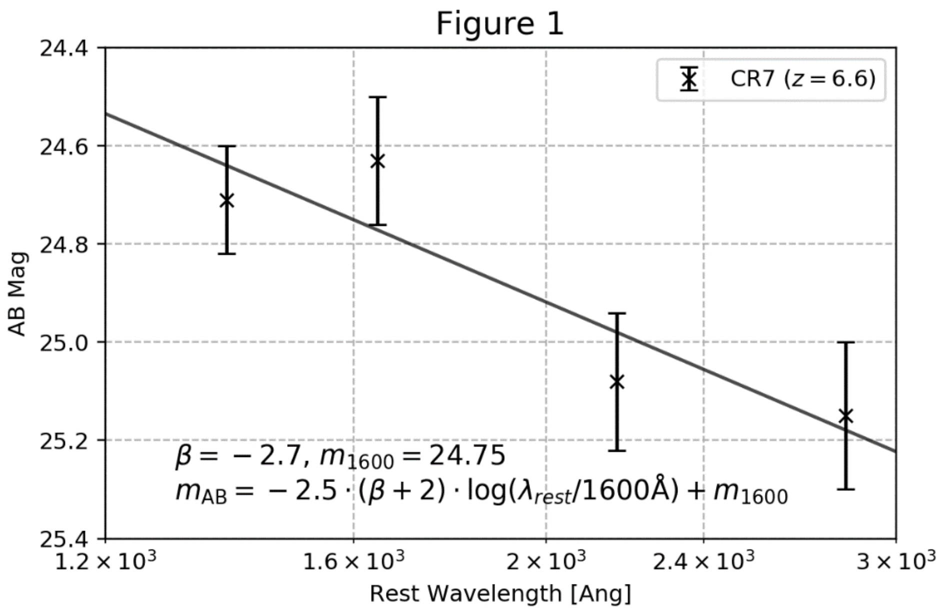
Here m_{1600} is the AB magnitude at rest-frame 1600Å (observed-frame $(1+z)\cdot 1600$ Å). After writing this out, we can fit the data with a linear function $f(\log\lambda) = a\log\lambda + b$, where the slope $a = -2.5(\beta+2)$. Least squares fitting result is $a = 1.7$, therefore $\beta = -2.7$.

(Note: It is not necessary for examinees to express m_{AB} in the same format as Equation 4. If an examinee derived the correct value of β, he or she will get full marks in this part.)

The best-fit UV continuum should be plot as a straight line on Figure 1. [beta: 3 marks]

Grading standard of Figure 1: (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is rest-frame wavelength and plotted in log scale; (3) [1 marks]vertical axis is AB magnitude and plotted reversely in linear scale (i.e. brighter magnitude is higher in the panel); (4) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (5) [1 mark]Data points and best-fit continuum are plotted properly; (6) [2 mark]Error bars of data points are presented properly;

Compared to the galaxy in (D1.1), CR7 shows smaller β value, indicating its UV continuum is bluer than typical galaxy. Therefore, there should be less dust content in CR7, and the correct answer should be [NO]. [2 marks]



1.2. UV slope and IRX[20marks]

(D1.2.1) [14marks]For the galaxy sample in Table 2, the IRX could be calculated as:

$$\text{IRX} = \log F_{FIR} - \log F_{1600} \ldots\ldots (5)$$

[Equation (5): 1 mark]

Therefore, the final dataset we need for plotting is:

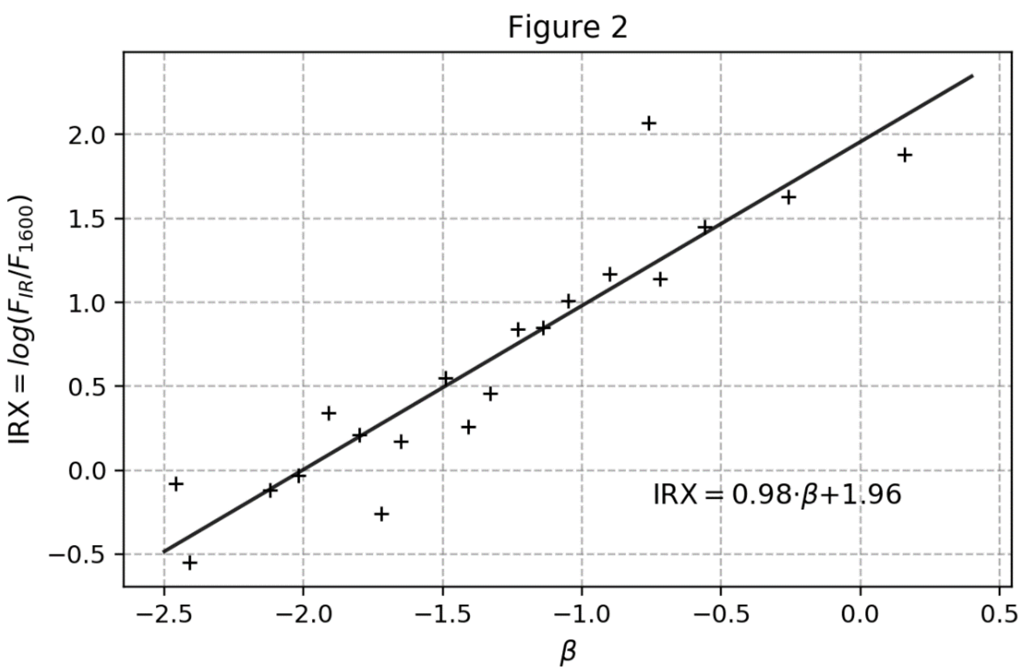
Galaxy Name	UV Slope β	IRX	Galaxy Name	UV Slope β	IRX
NGC 4861	-2.46	-0.08	NGC 7250	-1.33	0.46
Mrk 153	-2.41	-0.55	NGC 7714	-1.23	0.84
Tol 1924-416	-2.12	-0.12	NGC 3049	-1.14	0.85
UGC 9560	-2.02	-0.03	NGC 3310	-1.05	1.01
NGC 3991	-1.91	0.34	NGC 2782	-0.9	1.17
Mrk 357	-1.8	0.21	NGC 1614	-0.76	2.07
Mrk 36	-1.72	-0.26	NGC 6052	-0.72	1.14
NGC 4670	-1.65	0.17	NGC 3504	-0.56	1.45
NGC 3125	-1.49	0.55	NGC 4194	-0.26	1.63
UGC 3838	-1.41	0.26	NGC 3256	0.16	1.88

The plot is shown as <u>*Figure 2*</u>. Best-fit function is:

$$\text{IRX} = 0.98 \cdot \beta + 1.96 \ldots\ldots (6)$$

[Equation (6): 3 marks; Figure 2: 10 marks]

which should be plotted as a straight line on Figure 2.



<u>Grading standard of Figure 2:</u> (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is β, vertical axis is IRX, and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4)

[5 marks]Data points and best-fit function are plotted properly; (5) [1 mark]Best-fit equation is noted correctly on the same page of Figure 2;

(D1.2.2) [6 marks]With the IRX-β relation we derived above, we can calculate the difference between observed IRX and expected value (inferred by β) of each galaxy：

Name	IRX_{obs}	IRX_{exp}	ΔIRX	Name	IRX_{obs}	IRX_{exp}	ΔIRX
NGC 4861	-0.08	-0.45	0.37	NGC 7250	0.46	0.66	-0.20
Mrk 153	-0.55	-0.40	-0.15	NGC 7714	0.84	0.75	0.09
Tol 1924-416	-0.12	-0.12	0.00	NGC 3049	0.85	0.84	0.01
UGC 9560	-0.03	-0.02	-0.01	NGC 3310	1.01	0.93	0.08
NGC 3991	0.34	0.09	0.25	NGC 2782	1.17	1.08	0.09
Mrk 357	0.21	0.20	0.01	NGC 1614	2.07	1.22	0.85
Mrk 36	-0.26	0.27	-0.53	NGC 6052	1.14	1.25	-0.11
NGC 4670	0.17	0.34	-0.17	NGC 3504	1.45	1.41	0.04
NGC 3125	0.55	0.50	0.05	NGC 4194	1.63	1.71	-0.08
UGC 3838	0.26	0.58	-0.32	NGC 3256	1.88	2.12	-0.24

Therefore, the dispersion of IRX is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} \Delta \mathrm{IRX}_i^2}{N-1}} = 0.28 \text{ dex} \ldots\ldots (7)$$

[Equation (7): 6 marks]

1.3. IRX and A_{1600}:[20marks]

(D1.3.1) [6marks]Base on the two assumptions we made in Question (3), we can simplify F_{FIR}/F_{1600} as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1 \ldots\ldots (8)$$

$$\therefore A_{1600} = 2.5\log(1 + 10^{\mathrm{IRX}}) \ldots\ldots (9)$$

[Equation (9): 6 marks]

(D1.3.2) [12marks]Now we can calculate A_{1600} of all the galaxy in Table 2 with Equation 9:

Name	β	A_{1600}	Name	β	A_{1600}
NGC 4861	-2.46	0.66	NGC 7250	-1.33	1.47
Mrk 153	-2.41	0.27	NGC 7714	-1.23	2.25
Tol 1924-416	-2.12	0.61	NGC 3049	-1.14	2.27
UGC 9560	-2.02	0.72	NGC 3310	-1.05	2.63
NGC 3991	-1.91	1.26	NGC 2782	-0.9	3.00

Mrk 357	-1.8	1.05	NGC 1614	-0.76	5.18
Mrk 36	-1.72	0.48	NGC 6052	-0.72	2.93
NGC 4670	-1.65	0.99	NGC 3504	-0.56	3.66
NGC 3125	-1.49	1.64	NGC 4194	-0.26	4.10
UGC 3838	-1.41	1.13	NGC 3256	0.16	4.71

Then we can plot $A_{1600} - \beta$ relation as *<u>Figure 3</u>*. The best-fit equation is:

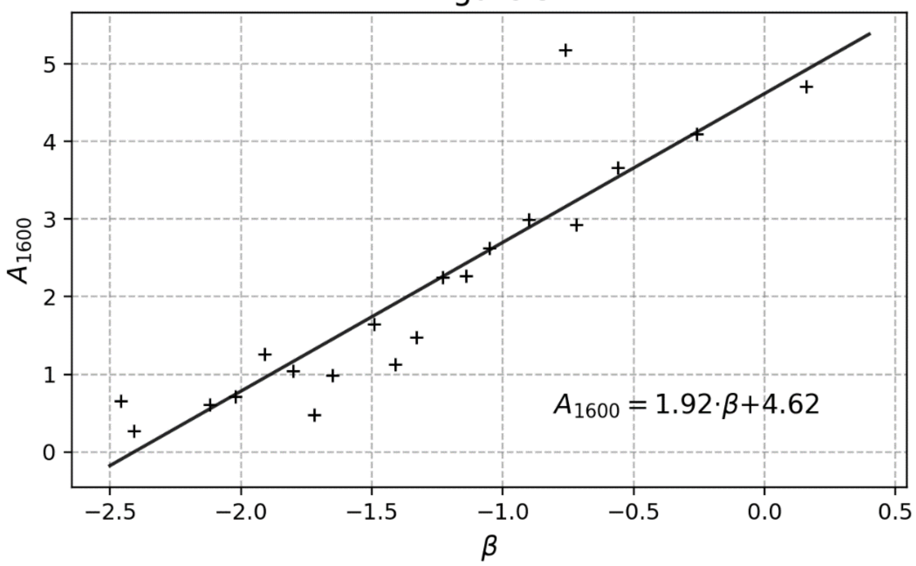
$$A_{1600} = 1.92 \cdot \beta + 4.62 \ldots\ldots (10)$$

[Equation (10): 2 marks; Figure 3: 10 marks]

<u>Grading standard of Figure 3:</u> (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is β, vertical axis is A_{1600}, and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4) [5 marks]Data points and best-fit function are plotted properly; (5) [1 mark]Best-fit equation is noted correctly on the same page of Figure 3;

(D1.3.3) [2marks]If Equation 10 is correct, then for a dust-free galaxy, the dust attenuation A_{1600} should equal to 0. Therefore, we can derive $\beta_0 \approx -2.4$, which is the expect UV continuum slope of a "dust-free" galaxy.

Figure 3



(Note: The β_0 result here is conflict with some observations, for example, the UV slope of CR7 which we just derived. In fact, the linear model here is not necessarily correct and it is just a simplification of complex model: we have made two very rough assumption to derive the $A_{1600} -$ IRX relation, and other physical parameters of galaxy, e.g. metallicity, will also influence the overall $A_{1600} - \beta$ relation.)

1.4. Extend to high-redshift universe:[12marks] (discarded)

(D1.4.1) [6marks]Since we have already known the upper limit of F_{FIR} of CR7, we only need to derive F_{1600} in order to calculate IRX. First of all, we need to translate the m_{1600} we got in (D 1.3) to $f_{(1+z)1600\text{Å}}$:

$$f_{\lambda_0(1+z)} = f_{\nu_0(1+z)} \cdot \frac{c}{[\lambda_0(1+z)]^2} = \frac{c}{[\lambda_0(1+z)]^2} \cdot 10^{-0.4m_{1600}} \cdot 3631\ \text{Jy} \dots\dots (11)$$

and we will derive $f_{\lambda_0(1+z)} = 9.3 \times 10^{-13}\text{J} \cdot \text{s}^{-1}\text{m}^{-3}$ Therefore:

$$F_{1600} = (1+z) \cdot \lambda_0 \cdot f_{\lambda_0 \cdot (1+z)} = 1.1 \times 10^{-18}\text{J} \cdot \text{s}^{-1}\text{m}^{-2} \dots\dots (12)$$

Thus, the upper limit of *CR7*'sIRX should be:

$$\text{IRX}_{CR7} = \log \frac{F_{FIR}}{F_{1600}} = -0.87 \dots\dots (13)$$

[Equation (11), (12), (13): 2 marks respectively]

(D1.4.2) [6marks]Given the UV continuum slop of *CR7* (-2.7), we can infer its expected IRX from the $\text{IRX} - \beta$ relation in local universe:

$$\widehat{\text{IRX}}_{CR7} = 0.98 \cdot \beta_{CR7} + 1.96 = -0.69 \dots\dots (14)$$

The upper limit of observed IRX is 0.18 dex lower than the value predicted by local relation. However, we have already known the local relation has a dispersion of 0.28 dex, which is larger than the 0.18 dex difference. This indicates that the current IRX upper limit is not tight enough (in other words, current observation is not deep enough) to show any difference of *CR7* from local galaxies on the diagram of IRX$-\beta$. Therefore, the correct answer should be [NO].

[Equation (14): 2 marks; two IRX difference: 2 marks; [NO]: 2 marks]

(D2)**Compact Object in a Binary System** **[75 points]**

Astronomers discovered an extraordinary binary system in the constellation of Auriga during the course of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). In these questions, you will try to analyze the data and recreate their discovery by yourself.

The research team is aiming to find compact stars in binary systems using the radial velocity (RV) technique. They examined archival APOGEE spectra of "single" stars and measured the variation of their RV. Among ~200 stars with the highest accelerations, researchers searched for periodic photometric variations in data from the All-Sky Automated Survey for Supernovae (ASAS-SN) that might be indicative of transits, ellipsoidal variations or starspots. After this process, they spotted a star named 2M05215658+4359220, with large variation in RV and photometric variability.

2.1. The following table presents the radial velocity measurements of 2M05215658+4359220 during three epochs of APOGEE spectroscopic observation. Here we assume the variation of its RV is due to the existence of an unseen companion. The proper motion of the stars can be ignored.

Table 3. APOGEE Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
56204.9537	-37.417	0.011
56229.9213	34.85	0.01
56233.8732	42.57	0.01

(D2.1.1) (6 points) Use the data to obtain a rough estimate of the apparent **maximum acceleration:**

$$a_{max} = \left.\frac{\Delta RV}{\Delta t}\right|_{max}, \text{ unit: km/s/day}$$

of the star.

(D2.1.2) (9 points) Estimate the **mass** of its unseen companion with reasonable models.

2.2. After discovering this peculiar star, astronomers conducted follow-up observations using the 1.5m Tillinghast Reflector Echelle Spectrograph (TRES) at the Fred Lawrence Whipple Observatory (FLWO) located on Mt. Hopkins in Arizona, USA. The following table presents the RV measurements using this instrument:

Table 4. TRES Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
58006.9760	0	0.075
58023.9823	-43.313	0.075
58039.9004	-27.963	0.045
58051.9851	10.928	0.118
58070.9964	43.782	0.075
58099.8073	-30.033	0.054
58106.9178	-42.872	0.135
58112.8188	-44.863	0.088
58123.7971	-25.81	0.115

58136.6004	15.691	0.146
58143.7844	34.281	0.087

(D2.2.1) (14 points) **Plot** the diagram of RV variation (measured with TRES) versus time on your graph paper as Figure 4. Connect all your data points with a **sinusoidal function**. **Estimate** the orbital period (P_{orb}) and radial velocity semi-amplitude (K) from your plot.

(D2.2.2) (4 points)If the star is rotating in circular orbit, show the **minimum value** of the orbital radius (r_{orb}) of the star in unit of both $R_{\odot}$ and au.

(D2.2.3) (7 points)The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2}$$

where the subscript "1" represents the primary star and "2" represents its companion. The parameter i_{orb} is the orbital inclination of the binary system. Calculate the **mass function of this system** in unit of $M_{\odot}$.

2.3. Based on a detailed analysis on APOGEE, TRES spectra and GAIA parallax measurements, astronomers derived the following stellar parameters:

Table 5. Selected Physical Properties of 2M05215658+4359220

Effective Temperature T_{eff} (K)	Surface Gravity $\log g$ $(\mathrm{cm/s^2})$	Parallax π (mas)	Radial Rotation Velocity $v \sin i_{rot}$ (km/s)	Bolometric Flux F $(\mathrm{J\ s^{-1}m^{-2}})$
4890 ± 130	2.2 ± 0.1	0.272±0.049	14.1±0.6	(1.1±0.1)×10^{-12}

Photometric observations indicate that the period of its light curve is identical to orbital period, thus we may assume that rotation period satisfies $P_{rot} = P_{orb} \equiv P$, and the inclination satisfies $i_{orb} = i_{rot} \equiv i$.

(D2.3.1) (16 points)**Calculate** the sine of inclination angle ($\sin i$), radius (R_1, in unit of $R_{\odot}$), luminosity (L_1, in unit of $L_{\odot}$), as well as mass (M_1, in unit of $M_{\odot}$) of the visible star. Please **include** the uncertainty into your results.

(D2.3.2) (4 points)**Choose** the correct type of this star from the following options: (1) Blue Giant (2) Blue Dwarf (3) Red Giant (4) Red Dwarf (5) Main-Sequence Star (6) White Dwarf.

(D2.3.3) (10 points)Based on the mass function $f(M_1, M_2)$of binary system, **plot** out the rough relation of M_2 (as vertical axis) and M_1 (as horizontal axis) on your graph paper as Figure 5. Here you should adopt the $\sin i$, $\sin i + \Delta \sin i$and $\sin i - \Delta \sin i$derived in (D2.3.1), and plot out three functions **respectively**.

(D2.3.4) (5 points)Draw a vertical **shadowed region** of $[M_1 - \Delta M_1, M_1 + \Delta M_1]$, as well as two horizontal **dashed line** showing the maximum mass of white dwarf and neutron star, on your *Figure 5*. What is the possible mass of the invisible companion, and what kind of celestial object could it be?

Solution:

2.1. APOGEE RV Measurements[15 marks]

(D2.1.1)[6 marks]Table 3 gives 3 epochs of RV measurements from MJD 56204 to 56233, and the maximum acceleration occurred during the first and the second observation:

$$a_{max} = \frac{RV_2 - RV_1}{t_2 - t_1} = 2.89\ \mathrm{km/(s \cdot d)} \ldots\ldots (15)$$

(D2.1.2)[9 marks]An easy approach to estimate the companion mass is consider the force exerted on the star:

$$F = M_1 a = \frac{GM_1M_2}{r^2} \Rightarrow M_2 = \frac{ar^2}{G} \ldots\ldots (16)$$

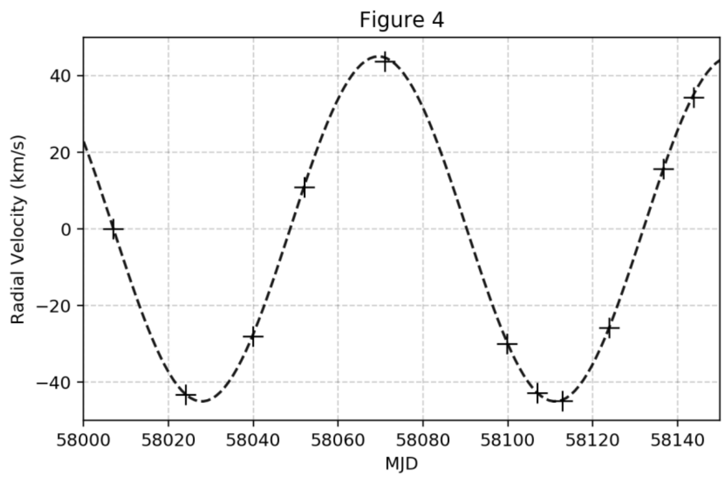
Therefore, we need to estimate a and r properly. Here we can assume the acceleration of the primary star as$a = a_{max}$, and the distance between the two star $r = (v_2 - v_1) \cdot (t_2 - t_1)/2$, and we can derive:

$$M_2 \sim \frac{(v_2 - v_1)^3 (t_2 - t_1)}{4G} = 1.5\ M_{\odot} \ldots\ldots (17)$$

(*Note: This is only a rough estimate. If examinees derive a* M_2*within the range of* $0.5M_{\odot} \sim 3\ M_{\odot}$[4 marks]*with clear and proper assumption*[5 marks]*, then his/her solution should be considered correct.)*

2.2. TRES RV Measurements[25 marks]

(D2.2.1)[14 marks]Given the data in Table 4, we can plot out all the TRES RV measurements, with a sinusoidal function connecting all the points:



<u>Grading standard of Figure 4:</u> (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is MJD, vertical axis is RV (in unit of km/s), and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4) [6 marks]Data points and sinusoidal function are plotted properly.

The best-fit parameters of period an RV semi-amplitude are:

$$P_{orb} = 83.26 \pm 0.02\ \mathrm{d};\ \ K = 45.02 \pm 0.03\ \mathrm{km/s} \ldots\ldots (18)$$

(error bars are reported here for benefit of jury. They do not carry any points)

[2 marks for each parameter]

Answers within the range of $P_{orb} \in [81, 85]$d and $K \in [44, 46]$ km/s could be considered correct.

(D2.2.2)[4 marks]The orbital radius satisfies that:

$$2\pi R_{orb} = P_{orb} \cdot \frac{K}{\sin i_{orb}} \Rightarrow R_{orb} > \frac{P_{orb}K}{2\pi} = 74\ R_{\odot} = 0.34\ \text{AU} \dots\dots (19)$$

(D2.2.3)[7 marks]The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2} \dots\dots (20)$$

This mass function is actually derived from Newton's law (force balance):

$$\frac{GM_1M_2}{D^2} = M_1 \left(\frac{2\pi}{P}\right)^2 R_{orb} \dots\dots (21)$$

And the relation between orbital radius and binary separation (D):

$$R_{orb} = \frac{M_2}{M_1 + M_2} D = \frac{P_{orb}K}{2\pi \sin i_{orb}} \dots\dots (22)$$

Therefore, Equation 20 could be rewritten as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2} = \frac{K^3 P_{orb}}{2\pi G} \approx 0.79\ M_{\odot} \dots\dots (23)$$

[Equation (21), (22): 2 marks respectively; Equation (23) and the result: 3 marks]

2.3. Stellar Parameters[35 marks]

(D2.3.1)[16 marks]The first (and the most direct) thing we could solve from the given parameter table is the distance of this system, which is:

$$d = \frac{1}{\pi} = 3.68\ \text{kpc} \dots\dots (24)$$

And the uncertainty of d should be:

$$\frac{\Delta d}{d} = \sqrt{\left(-\frac{\Delta\pi}{\pi}\right)^2} \Rightarrow \Delta d = \frac{\Delta\pi}{\pi} \cdot d = 0.66\ \text{kpc} \dots\dots (25)$$

Therefore, the luminosity of the star should be:

$$L_1 = 4\pi d^2 \cdot F = 464\ L_{\odot} \dots\dots (26)$$

And the uncertainty is:

$$\frac{\Delta L_1}{L_1} = \sqrt{\left(\frac{\Delta F}{F}\right)^2 + \left(\frac{2\Delta d}{d}\right)^2} \Rightarrow \Delta L_1 = 173\ L_{\odot} \dots\dots (27)$$

By using Stefan-Boltzmann law, we can calculate the radius of the star:

$$R_1 = \sqrt{\frac{L_1}{\sigma T^4 \cdot 4\pi}} = 30\ R_\odot \dots\dots (28)$$

With an uncertainty of:

$$\Delta R_1 = R_1 \cdot \sqrt{\left(-2\frac{\Delta T}{T}\right)^2 + \left(\frac{1}{2}\frac{\Delta L_1}{L_1}\right)^2} = R_1 \cdot \sqrt{\left(-2\frac{\Delta T}{T}\right)^2 + \left(\frac{1}{2}\frac{\Delta F}{F}\right)^2 + \left(\frac{\Delta d}{d}\right)^2} = 6\ R_\odot \dots\dots (29)$$

Combined with the observed rotational velocity, we can calculate the sine of inclination angle:

$$\sin i = \frac{P \cdot v \sin i}{2\pi R_1} = 0.77 \dots\dots (30)$$

And the uncertainty should be:

$$\Delta \sin i = \sin i \cdot \sqrt{\left(-\frac{\Delta R_1}{R_1}\right)^2 + \left[\frac{\Delta(v \sin i)}{v \sin i}\right]^2} = 0.15 \dots\dots (31)$$

Now we can calculate the mass M_1. First of all, we need to translate the surface gravity from log scale to linear scale:

$$g = 10^{\log g} = 1.58\ \mathrm{m/s^2} \dots\dots (32)$$

With an uncertainty of:

$$\Delta g = g \ln 10 \cdot \Delta(\log g) = 0.36\ \mathrm{m/s^2} \dots\dots (33)$$

Therefore, M_1 should be:

$$M_1 = \frac{g R_1^2}{G} = 5.2\ M_\odot \dots\dots (34)$$

And the uncertainty is:

$$\Delta M_1 = M_1 \sqrt{\left(\frac{\Delta g}{g}\right)^2 + \left(2\frac{\Delta R_1}{R_1}\right)^2} = 2.3\ M_\odot \dots\dots (35)$$

[For each quantity (sin*i*, R_1, M_1, L_1):

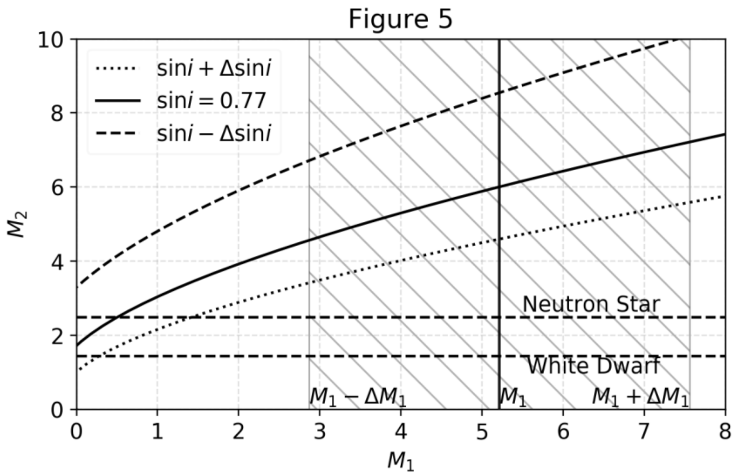
deduction: 2 marks; exact value: 1 mark; uncertainty: 1 mark]

(D2.3.2)[4 marks]From the temperature and the luminosity, we can find that the primary star should be a (3) Red Giant.

(D2.3.3)[10 marks]Since we have derived the binary system mass function, with a given range of M_1, we can calculate the corresponding range of M_2 and hence make a plot of $M_2 - M_1$ (Figure 5). Detailed standards of grading the plot are in the solution of (D2.3.4).

(D2.3.4)[5 marks]The vertical shadowed zone should start at 2.9 $M_\odot$ and end at 7.5 $M_\odot$, while the maximum mass of white dwarf is $1.4M_\odot$and the maximum mass of neutron star should be around 2.4~2.6 $M_\odot$(here we adopt 2.5 $M_\odot$ on the plot). From Figure 5, we can readout that the possible mass of the companion is roughly $6^{+4}_{-2.5}M_\odot$. Since we didn't receive any optical radiation from the companion, as well as M_2 has already exceeded the maximum mass of neutron star, the invisible companion in this binary system could be a **stellar-mass black hole**, or some unknown compact object that current theory hasn't predicted.

[Possible Mass: 2 marks; Type: 2 marks]



(Note: Examinee's answer is not required to give an uncertainty range of M_2: if the answer is within the possible range of $6^{+4}_{-2.5}M_\odot$, the answer will be considered correct. In addition to this, examinee don't need to include "unknown compact object" in his/her answer while addressing the possible type of the companion: an answer like "black hole" or "stellar-mass black hole" is enough for receiving full grades.)

Grading standard of Figure 5: (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is M_1, vertical axis is M_2 (both in unit of $M_\odot$), and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4) [3 mark]All of the three mass functions are plotted properly with obvious and correct labels/legends; (5) [1 mark]Vertical shadowed region of possible M_1 range is plotted correctly; (5) [2 marks]Two horizontal dashed lines are plotted at correct positions with clear and correct labels.